The role of vector interactions in quark matter using the NJL model

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Outline

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Introduction : the NJL model

- Effective quantum field theory (relativistic)
- Approximation of QCD in the non-perturbative domain
- Quark degrees of freedom
- Reproduces the flavor symmetries of QCD:

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_B$$

Spontaneous breaking of $SU(N_f)_A$

↙ Dynamical generation of fermion masses

↘ Goldstone mechanism

- (T, μ) ↗ : symmetry restoration ⟹ phase transition(s)

The NJL Lagrangian

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m} + \hat{\mu}\gamma_0)\psi}_{\text{Quarks}} + \overbrace{\bar{\psi}\gamma^\mu \frac{\lambda_a}{2} A_\mu{}^a \psi}^{\text{Quark/gluon interaction}} - \underbrace{\frac{1}{4} F_{\mu\nu}{}^a F^{\mu\nu}{}_a}_{\text{Gluons}}$$

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m} + \hat{\mu}\gamma_0)\psi + \sum_C G^C (\bar{\psi}\Gamma^C \psi)^2 + \mathcal{L}_{'t\ Hooft}$$

- Interaction parametrized by several coupling constants associated to the different channels considered: G_S, G_ρ, G_ω, K
- Total interaction must preserve the flavor symmetries
- $U(1)_A$ symmetry group broken by the 't Hooft term (anomaly)

The mean field approximation

- Assume small fluctuations of the fields around a mean value

$$\hat{\mathcal{O}} = \langle \hat{\mathcal{O}} \rangle + \delta \hat{\mathcal{O}}$$

- For scalar interactions : $G_S(\bar{\psi}\psi)^2 \approx \underbrace{2G_S\langle\bar{\psi}\psi\rangle\bar{\psi}\psi}_{\text{Mass modification}} + G_S\langle\bar{\psi}\psi\rangle^2$

- For vector interactions $G_V(\bar{\psi}\gamma_0\psi)^2 \approx \underbrace{2G_V\langle\psi^\dagger\psi\rangle\psi^\dagger\psi}_{\text{Chemical potential modification}} + G_V\langle\psi^\dagger\psi\rangle^2$

- With correct flavor factors following $SU(N_f = 3)$ algebra:

$$m_i = m_{i\,0} - 4G_S\,\langle\bar{\psi}_i\psi_i\rangle + 2K\langle\bar{\psi}_j\psi_j\rangle\langle\bar{\psi}_k\psi_k\rangle$$

$$\tilde{\mu}_i = \mu_i - \frac{4}{3}G_\omega(n_i + n_j + n_k) - \frac{4}{3}G_\rho(2n_i - n_j - n_k)$$

$i, j, k = u, d, s$

Model parameters

- 4 coupling constants : $G_S\,, G_\omega\,, G_\rho\,, K$
- 3 bare masses for the quarks : $m_{u\,0}\,, m_{d\,0}\,, m_{s\,0}$
- 1 momentum cutoff Λ

} 8 (7) parameters

- Fitted to mesonic data in the vacuum :

$m_\pi, f_\pi, m_K, m_{\eta'}, -\langle\bar{\psi}\psi\rangle^{\frac{1}{3}}$ ⟶ 5 experimental input

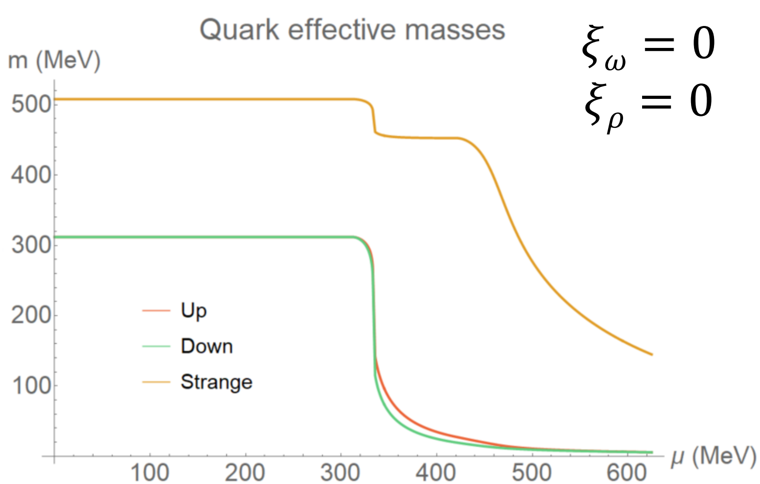
- We keep 2 free parameters: $\xi_\omega = \frac{G_\omega}{G_S}\;,\;\xi_\rho = \frac{G_\rho}{G_S}$

The phase transition

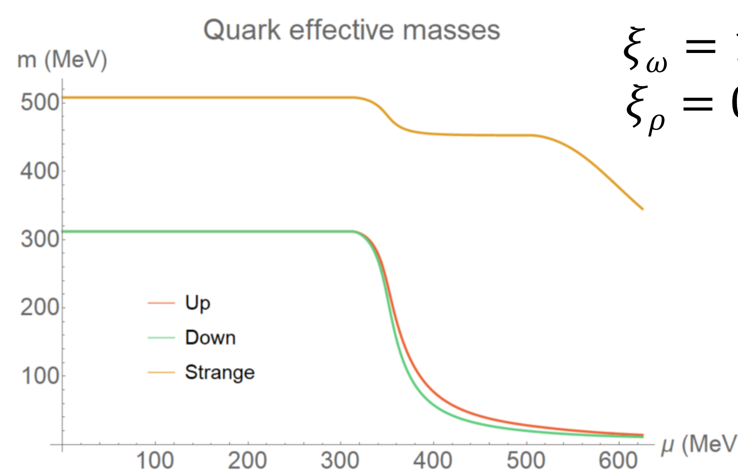
- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

$$\Longrightarrow \begin{cases} \mu_u = \mu + \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \mu - \frac{1}{3}\mu_e \end{cases}$$

- Chiral symmetry restoration



$\xi_\omega = 0$
$\xi_\rho = 0$



$\xi_\omega = 1$
$\xi_\rho = 0$

The phase transition

- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

$$\Longrightarrow \begin{cases} \mu_u = \mu + \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \mu - \frac{1}{3}\mu_e \end{cases}$$

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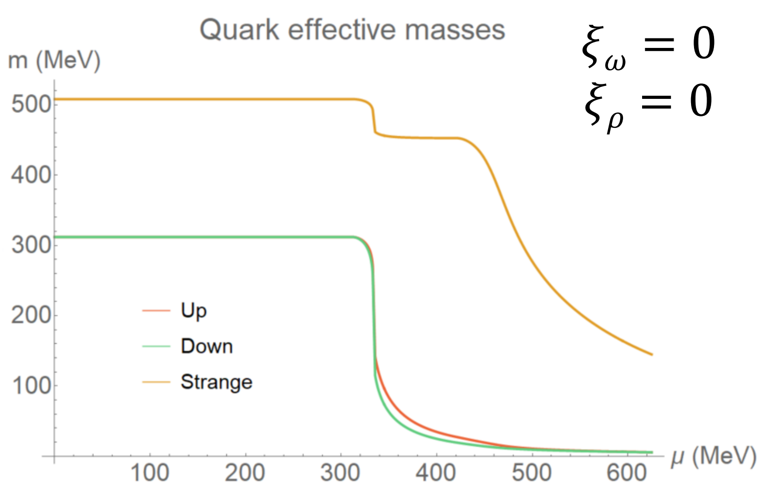

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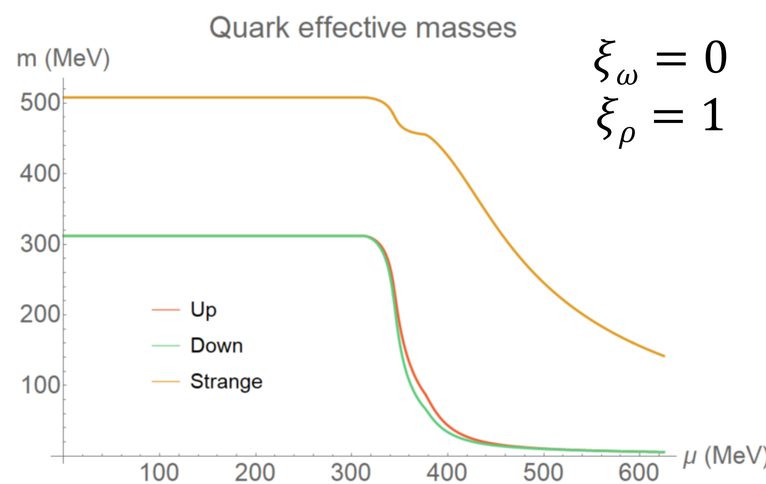


$\xi_\omega = 0$
$\xi_\rho = 1$

The equation of state

Fermi pressure of a free gas of quasi-quarks

Additional contribution from vector interactions

$$P_{tot} = P_{quarks} - B_{eff} + P_{vector} + P_{leptons}$$

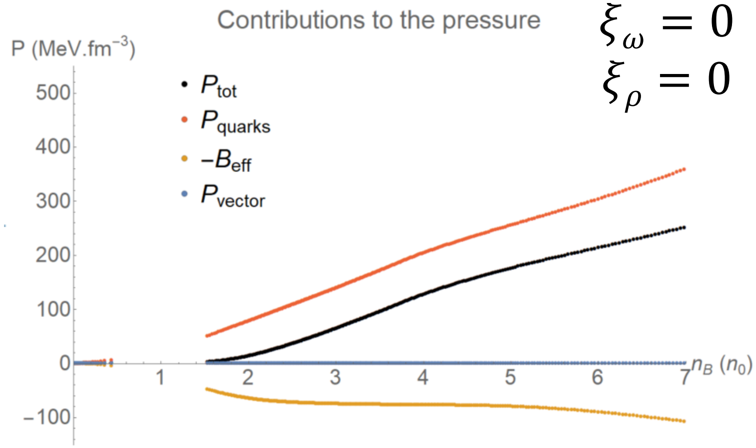
Effective " bag " pressure

Fermi pressure of electrons (+ muons)

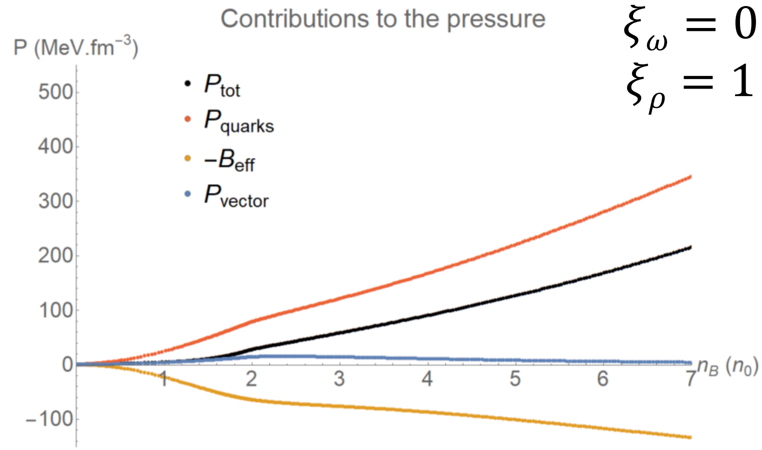
$$P_{vector} = \frac{2}{3} G_{\omega}(n_u + n_d + n_s)^2 + G_{\rho}(n_u - n_d)^2 + \frac{1}{3} G_{\rho}(n_u + n_d - 2n_s)^2$$

- ω interactions couple to the total baryonic density of the system (symmetric in flavor)
- ρ interactions couple to the isospin and flavor hypercharge densities (asymmetric in flavor)

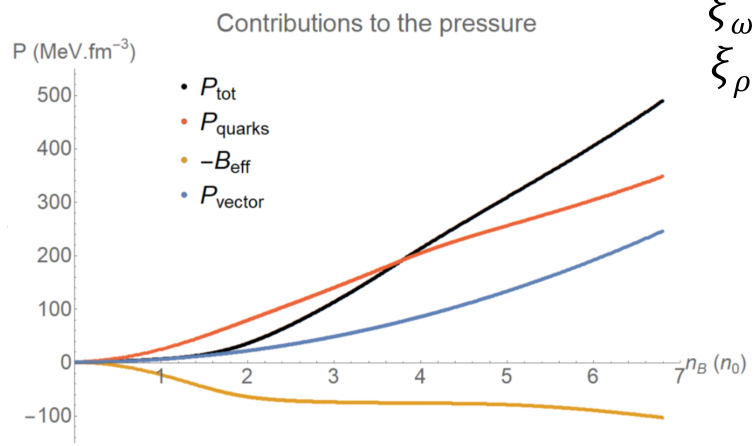
The equation of state



$\xi_\omega = 0$
$\xi_\rho = 0$

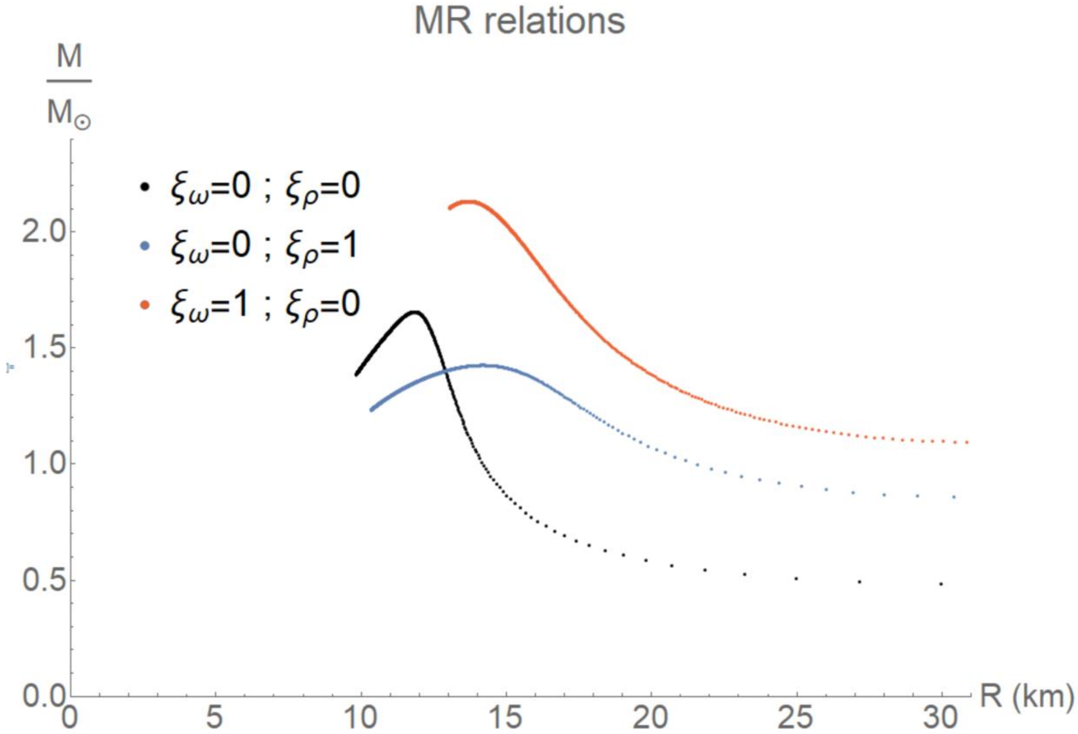


$\xi_\omega = 0$
$\xi_\rho = 1$



$\xi_\omega = 1$
$\xi_\rho = 0$

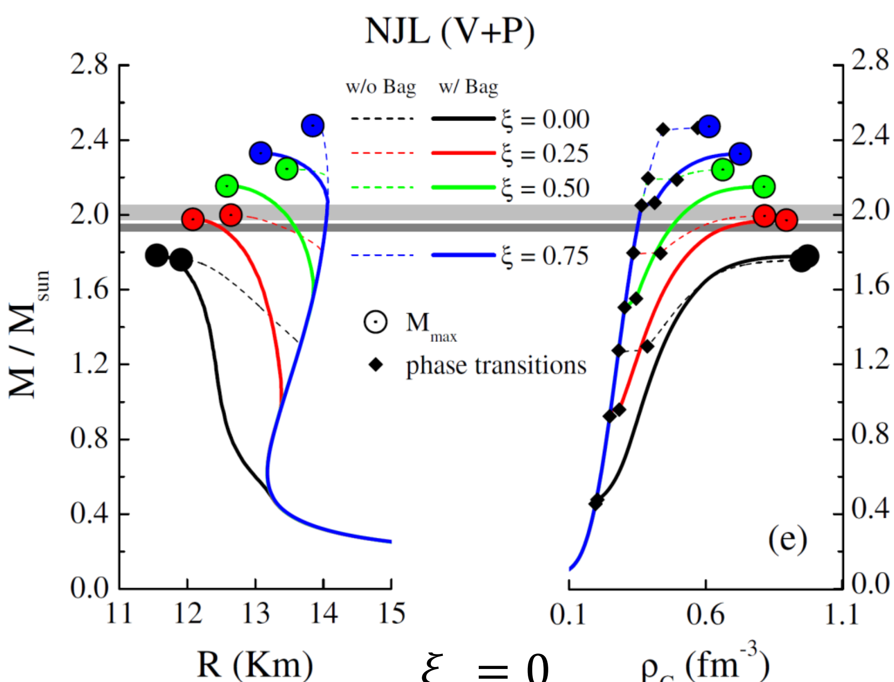
Mass-radius relations



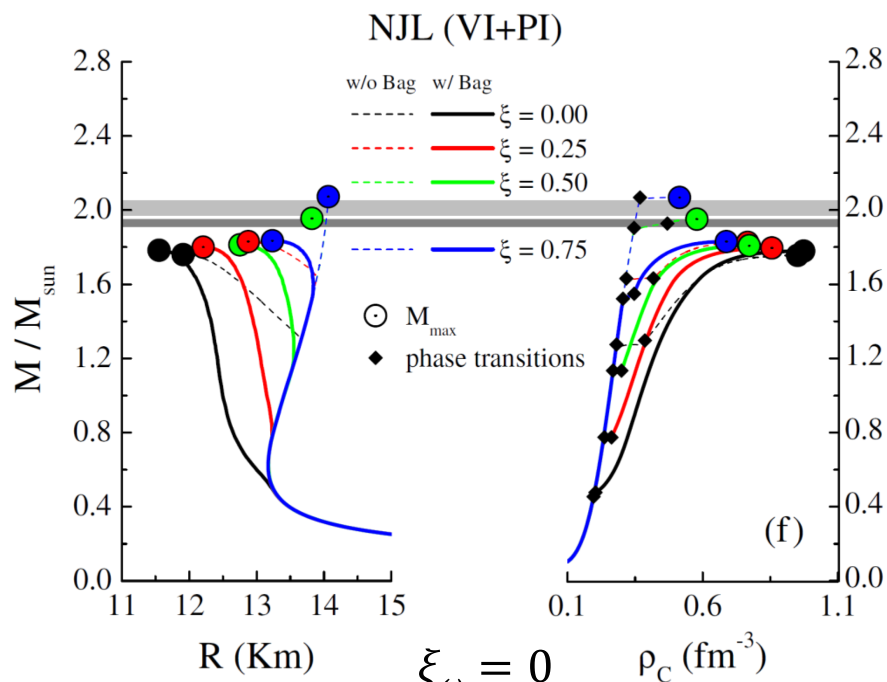
- ω interactions make the EOS stiffer, increasing the maximum mass of the sequence
- ρ interactions modify the flavor content at moderate density, which reduces the pressure and increases the radii of the stars

Hybrid star EOS

- Quark-hadron hybrid stars using the RMF NL3$\omega\rho$ model for the nuclear phase
- Two phases connected by a 1st order transition (Maxwell's construction)



$\xi_\rho = 0$
$\xi = \xi_\omega$



$\xi_\omega = 0$
$\xi = \xi_\rho$

Pereira et al. , arxiv.org/abs/1610.06435

Conclusion

- Vector-isoscalar (ω) interactions hardens the EOS, allowing to reach the $2M_{\odot}$ threshold

- Vector-isovector (ρ) interactions affect the flavor content of the system, favoring symmetric matter

- Both reduce the size of the quark core in hybrid stars