Uncertainties in pasta observables of catalyzed neutron stars

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MAC Meeting

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March 19, 2021

Outline

- Baym, Bethe, and Pethick in 1971 predicted nuclei to “turn inside out, that is, for the neutron gas to exist as a lattice of droplets in a sea of nuclear matter”. *[Baym et al., Nuclear Physics, 1971]*

1. Introduction - **Pasta phases**

- Baym, Bethe, and Pethick in 1971 predicted nuclei to "turn inside out, that is, for the neutron gas to exist as a lattice of droplets in a sea of nuclear matter". *[Baym et al., Nuclear Physics, 1971]*
- In the CLD model, the cluster energy can be decomposed into bulk, surface, and Coulomb terms:

$$E_{cl} = e_{HM}(n_0, I)A + \underbrace{E_{surf+curv} + E_{Coul}.}_{\text{depend on nuclear shape}} \tag{1}$$

$\rightarrow$ At high density, before the CC transition, nuclei may exist in non-spherical shapes, collectively called nuclear pasta.

- Expressions of surface, curvature, and Coulomb energy densities:

$$\epsilon_{surf} = \frac{ud\sigma_s}{r_n}, \quad (2)$$

$$\epsilon_{curv} = \frac{ud(d-1)\sigma_c}{r_n^2}, \quad (3)$$

$$\epsilon_{Coul} = 2\pi(eY_p n_0 r)^2 u\eta_{Coul,d}(u). \quad (4)$$

[Ravenhall et al., Physical Review Letter, 1983]

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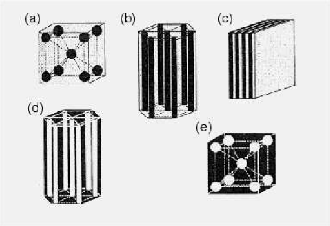
[Ravenhall et al., Physical Review Letter, 1983]

- 5 phases: spheres, cylinders, plates, tubes, bubbles

"inverted" configurations



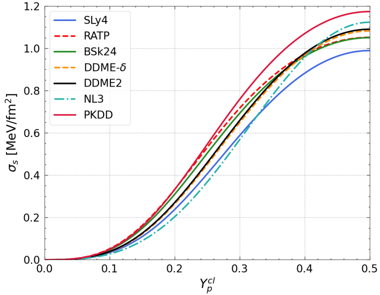
1. Introduction - **Surface and curvature tensions**

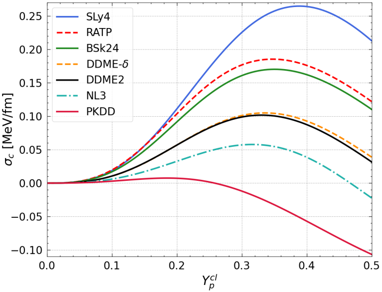
$$\sigma_s(Y_p) = \sigma_0 \frac{2^{p+1} + b_s}{Y_p^{-p} + b_s + (1 - Y_p)^{-p}}, \quad (5)$$

$$\sigma_c(Y_p) = \frac{\sigma_{0,c}}{\sigma_0} \alpha(\beta - Y_p)\sigma_s(Y_p). \quad (6)$$

- $\alpha = 5.5$
- 5 surface parameters:

$$\mathbf{S} = \{ \overbrace{p}^{\text{optimized}}, \underbrace{\sigma_0, \sigma_{0,c}, b, \beta}_{\text{fitted to AME2016}} \}$$





1. Introduction - **Equilibrium equations**

$$\begin{aligned}\Omega = {} & n_p m_p c^2 + (n_B - n_p) m_n c^2 \\ & + n_0 e_{HM}(n_0, I)\mathrm{f} + \epsilon_{surf} + \epsilon_{curv} + \epsilon_{Coul} \\ & + n_g e_{HM}(n_g, 1)(1 - \mathrm{f}) + \epsilon_e - \mu n_B.\end{aligned} \tag{7}$$

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- Minimizing Ω at fixed baryon density with respect to A, I, n_0, n_g $\rightarrow$ 4 equations:

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- Minimizing Ω at fixed baryon density with respect to A, I, n_0, n_g $\rightarrow$ 4 equations:
- Baryonic number conservation:

$$n_B = n_g + \frac{2n_p}{1 - \mathrm{I}}\left(1 - \frac{n_g}{n_0}\right). \quad (8)$$

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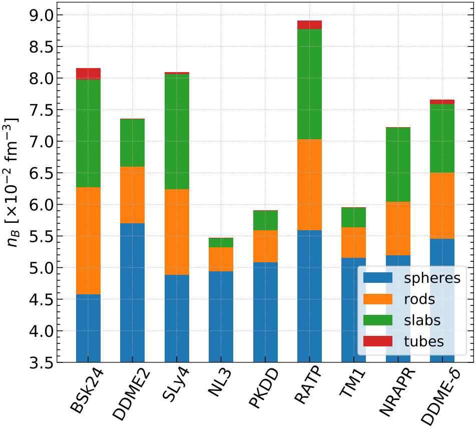
- Solve for $\{r_n, n_0, n_g, n_p, \mathrm{I}\}$
-

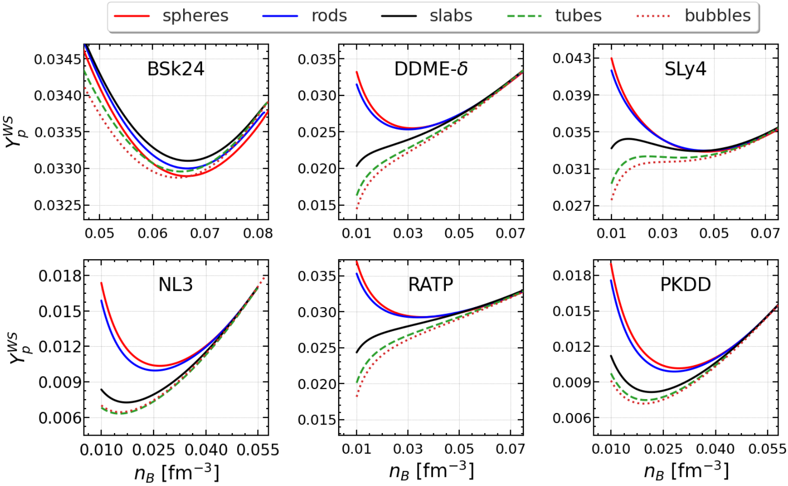
$$A = \frac{n_B - n_g}{1 - \frac{n_g}{n_0}} V_{WS}. \quad (9)$$

2. Model dependence - **Transition points**



	n_{sc}/n_{CC} ($\times 10^{-2}$fm^{-3})					
Model	This work	Pearson et al.	M&U	D&H	Vinas et al	Grill et al.
BSk24	4.6/8.15	5.0/8.1				
SLy4	4.9/8.1		6.1/8.1	-/7.6	-/7.3	
NL3	4.9/5.47					-/5.48
DD-ME2	5.7/7.35					6.11/7.35
DDME-δ	5.5/7.66					6.26/7.66

2. Model dependence - **Proton fraction**



- Prior:

Parameters	Min	Max
E_{sat} (MeV)	-17	-15
n_{sat} (fm^{-3})	0.15	0.17
K_{sat} (MeV)	190	270
Q_{sat} (MeV)	-1000	1000
Z_{sat} (MeV)	-3000	3000
E_{sym} (MeV)	26	38
L_{sym} (MeV)	10	80
K_{sym} (MeV)	-400	200
Q_{sym} (MeV)	-2000	2000
Z_{sym} (MeV)	-5000	5000
$m{*}_{sat}$	0.6	0.8
$\Delta m *_{sat} /m$	0.0	0.2
b	1	6

3. Bayesian inference

- Prior:

Parameters	Min	Max
E_{sat} (MeV)	-17	-15
n_{sat} (fm^{-3})	0.15	0.17
K_{sat} (MeV)	190	270
Q_{sat} (MeV)	-1000	1000
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K_{sym} (MeV)	-400	200
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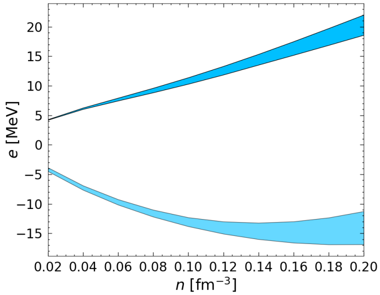
Z_{sym} (MeV)	-5000	5000
$m*_{sat}$	0.6	0.8
$\Delta m *_{sat} / m$	0.0	0.2
b	1	6

- Low density (LD) filter:



[Drischler et al., Physical Review C, 2016]

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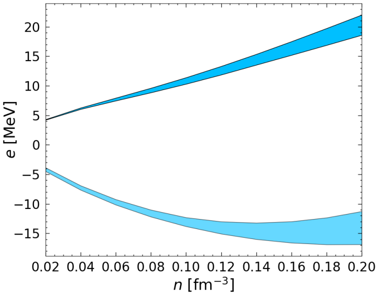
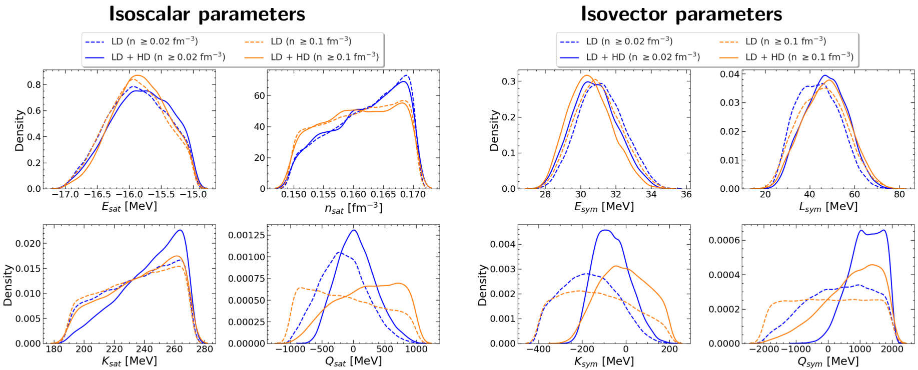


[Drischler et al., Physical Review C, 2016]

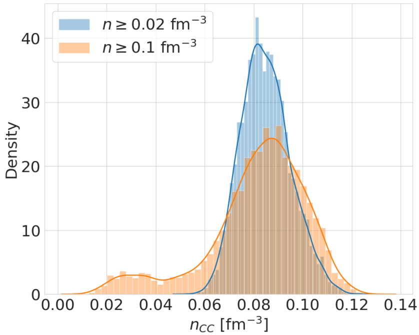
- High density (HD) filter:
  + $c_s/c < 1$.
  + $dP/d\rho > 0$.
  + $M_{max} \geq 1.97 M_{\odot}$.
  + Symmetry energy is positive at all densities.

3.1. Distributions of EoS parameters



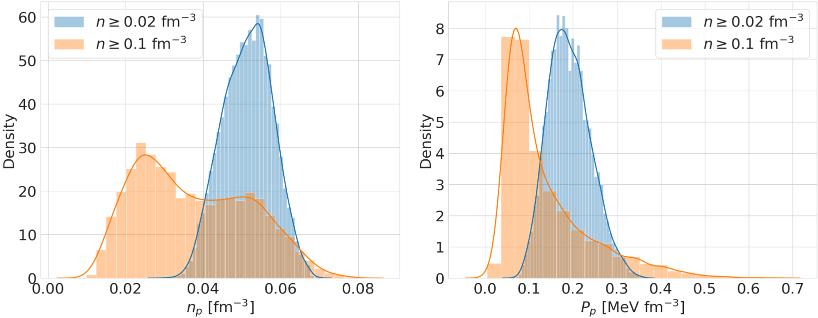
→ Filtering from 0.02 fm^{-3} constrains better the high-order parameters.

3.2. Crust-core transition



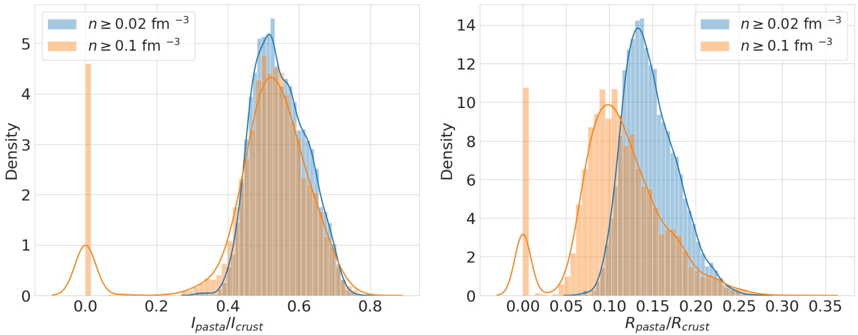
	LH+HD $n \geq 0.02$ fm^{-3}	LD+HD $n \geq 0.1$ fm^{-3}
n_{CC} (fm^{-3})	0.085 ± 0.010	0.082 ± 0.020
P_{CC} (MeVfm^{-3})	0.441 ± 0.099	0.297 ± 0.216

3.3. Pasta observables



→Very low density part of the equation of state has a drastic effect in the determination of the transition points.

	LH+HD $n \geq 0.02$ fm $^{-3}$	LD+HD $n \geq 0.1$ fm $^{-3}$
n_p (fm^{-3})	0.051 ± 0.006	0.038 ± 0.014
P_p (MeVfm^{-3})	0.192 ± 0.048	0.137 ± 0.098



	LH+HD $n \geq 0.02$ fm $^{-3}$	LD+HD $n \geq 0.1$ fm $^{-3}$
R_{pasta}/R_{crust}	0.148 ± 0.030	0.111 ± 0.051
I_{pasta}/I_{crust}	0.546 ± 0.074	0.495 ± 0.161
m_{pasta}/m_{crust}	0.550 ± 0.074	0.499 ± 0.162

n_p

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β
LD+HD ($n \geq 0.02$ fm^{-3})	-0.88	0.20	0.35	-0.28	0.03	0.35	-0.20	-0.27	0.20	-0.02	0.89	0.61	-0.75	-0.86
LD+HD ($n \geq 0.1$ fm^{-3})	-0.31	0.08	0.22	-0.17	0.00	0.29	-0.25	-0.61	0.60	-0.18	0.31	0.03	-0.15	-0.31
Prior	-0.23	0.02	0.10	-0.12	0.03	-0.03	0.08	-0.48	0.42	-0.07	0.24	0.10	-0.22	-0.20

P_p

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β
LD+HD ($n \geq 0.02$ fm^{-3})	-0.69	0.17	0.29	-0.20	0.01	0.37	-0.33	-0.38	0.24	-0.03	0.69	0.38	-0.52	-0.67
LD+HD ($n \geq 0.1$ fm^{-3})	-0.20	0.04	0.20	-0.14	-0.01	0.25	-0.26	-0.59	0.52	-0.17	0.19	-0.05	-0.05	-0.19
Prior	-0.06	-0.02	0.09	-0.09	0.02	0.13	0.07	-0.42	0.38	-0.06	0.05	-0.11	0.01	-0.04

I_{pasta}/I_{crust}

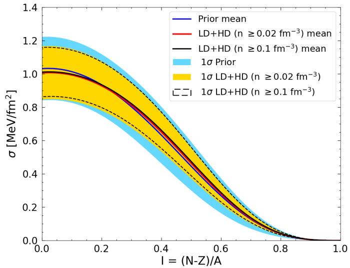
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β
LD+HD ($n \geq 0.02$ fm^{-3})	0.89	-0.27	-0.28	0.21	-0.05	-0.61	-0.30	0.09	0.15	-0.06	-0.88	-0.35	0.68	0.84
LD+HD ($n \geq 0.1$ fm^{-3})	0.30	-0.12	0.06	-0.26	0.15	-0.30	-0.05	0.25	-0.04	-0.04	-0.29	0.00	0.17	0.27
Prior	0.30	-0.00	0.08	0.01	0.08	0.15	-0.00	0.41	0.25	0.10	-0.32	-0.25	0.35	0.28

R_{pasta}/R_{crust}

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β
LD+HD ($n \geq 0.02$ fm^{-3})	0.75	-0.22	-0.21	0.19	-0.06	-0.46	-0.45	-0.10	0.22	-0.05	-0.75	-0.36	0.63	0.70
LD+HD ($n \geq 0.1$ fm^{-3})	0.34	-0.11	0.07	-0.21	0.06	-0.15	-0.26	-0.23	0.34	-0.15	-0.35	-0.20	0.32	0.32
Prior	0.40	0.01	0.08	-0.00	0.07	0.30	-0.01	0.03	0.44	0.03	-0.44	-0.42	0.53	0.36

4. Influence of surface parameters

* **Surface parameters** are optimized for each EoS.

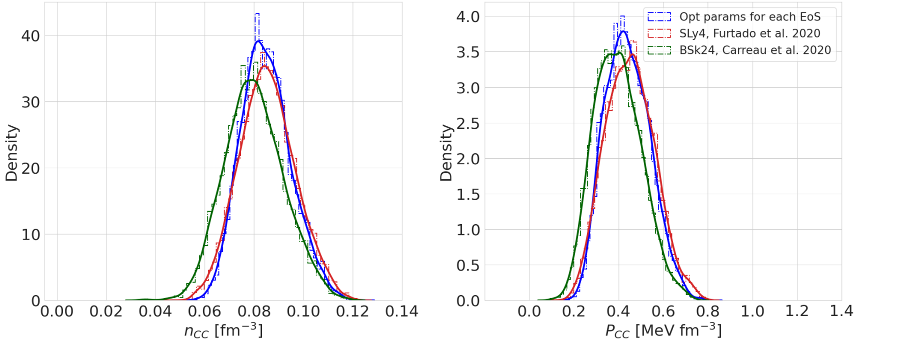


* What if we fix a set of surface parameters for all EoS?

2 sets of parameters are used:

- Carreau *et al.* 2020 (BSk24): p=3.0, $\sigma_0 = 0.98636$ MeV fm^{-2}, $b_s = 36.227$, $\sigma_{0c} = 0.09008$ MeV fm^{-1}, $\alpha = 5.5$, and $\beta = 1.1631$.
- Furtado *et al.* 2020 (SLy4): $\sigma_c = \frac{\sigma_{0c}}{\sigma_0}\alpha\sigma_s$. The optimal parameters are: p=3.4, $\sigma_0 = 0.99654$ MeV fm^{-2}, $b_s = 49.82$, $\sigma_{0c} = 0.061768$ MeV fm^{-1}, and $\alpha = 5.5$.

4.1. Crust-core transition



n_{CC}

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
All EoSs	-0.10	-0.05	0.16	-0.06	-0.03	-0.32	-0.68	-0.22	0.51	-0.17
SLy4	-0.24	-0.06	0.18	-0.14	-0.05	-0.32	-0.67	-0.20	0.52	-0.16
BSk24	-0.25	-0.09	0.17	-0.18	-0.04	-0.36	-0.63	-0.14	0.55	-0.20

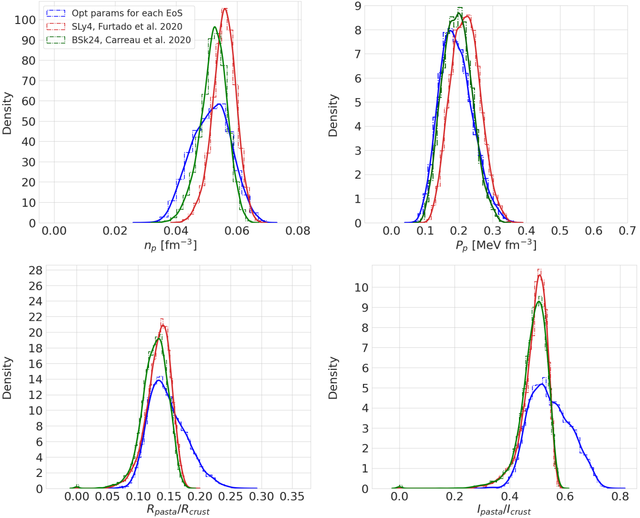
P_{CC}

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
All EoSs	-0.09	-0.01	0.13	-0.05	-0.03	-0.06	-0.63	-0.39	0.37	-0.06
SLy4	-0.25	-0.02	0.16	-0.13	-0.04	-0.07	-0.63	-0.37	0.41	-0.06
BSk24	-0.25	-0.06	0.15	-0.16	-0.04	-0.15	-0.64	-0.30	0.47	-0.11

→ Fixing the surface parameters does NOT significantly affect the crust-core transition points.

4.2. Pasta observables

...However, the affect on pasta quantities are non-negligible.



n_p

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
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SLy4	-0.20	0.13	0.28	-0.04	-0.08	-0.16	-0.57	-0.28	0.38	-0.04
BSk24	-0.24	0.07	0.26	-0.10	-0.07	-0.21	-0.57	-0.23	0.44	-0.09

P_p

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
All EoSs	-0.69	0.17	0.29	-0.20	0.01	0.37	-0.33	-0.38	0.24	-0.03
SLy4	-0.10	0.09	0.20	0.01	-0.09	0.00	-0.57	-0.38	0.32	-0.02
BSk24	-0.13	0.05	0.20	-0.03	-0.08	-0.06	-0.59	-0.33	0.39	-0.07

R_{pasta}/R_{crust}

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
All EoSs	0.75	-0.22	-0.21	0.19	-0.06	-0.46	-0.45	-0.10	0.22	-0.05
SLy4	-0.28	-0.25	0.02	-0.28	0.01	-0.14	-0.60	-0.30	0.44	-0.08
BSk24	-0.25	-0.28	0.00	-0.31	0.02	-0.25	-0.59	-0.20	0.50	-0.15

I_{pasta}/I_{crust}

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
All EoSs	0.89	-0.27	-0.28	0.21	-0.05	-0.61	-0.30	0.09	0.15	-0.06
SLy4	-0.44	-0.27	-0.01	-0.39	0.06	-0.22	-0.43	-0.08	0.44	-0.15
BSk24	-0.38	-0.28	-0.04	-0.39	0.07	-0.28	-0.36	0.01	0.45	-0.18

5. Conclusions

- Prediction of transition points and compositions are strongly model-dependent.
- Very low density part of the EoS has important effect in predicting transition points and pasta observables.
- In addition to EoS, the surface parameters are also crucial in determining the uncertainties of pasta observables.
- The uncertainties in the pasta quantities are underestimated if the uncertainty in surface parameters are not considered.